

Scattering Theory of Electro-

Magnetic waves :-

Interaction between electromagnetic waves and particles produce unique scattering patterns that are wavelength and particle size dependent.

As electromagnetic waves propagate through matter they interact with particles or inhomogeneities and locally perturb the local electron distribution.

Scattering of electromagnetic waves by particles can be treated by two theoretical frameworks:

Rayleigh scattering; that is applicable to small dielectric, non absorbing spherical particles, and

Mie scattering; that provides a general solution to scattering independent of particle size.

Rayleigh scattering is strongly dependent upon the size of the particles and wavelength of illuminating radiation. The intensity of Rayleigh scattered radiation increases rapidly as a ratio of particle

size to wavelength increases and is identical in forward and reverse direction.

The inherent scattering that radiation

undergoes passing through a pure gas is due to microscopic density fluctuations as the gas molecules move around, which are normally in scale for Rayleigh's model to apply.

This scattering mechanism is a major cause of the blue colour of the Earth's sky on a clear day.

$$m_d n_d \frac{dV_d}{dt} = \frac{e}{c} (Z_i n_i V_i - n_e V_e - Z_d n_d V_d) \times \vec{B}$$

$$- e (n_e + Z_d n_d - Z_i n_i) \vec{E}$$

$$- \nabla (P_e + P_{re} + P_i + P_{ri}) + \int d \nabla \psi \quad (4)$$

From eqs (A) R_α , the momentum gain of electrons during collisions with ion;

$$P_{ei} = m_e n_e V_{ei} (V_i - V_e)$$

and momentum gain of ion during collision with electron.

$$P_{ie} = m_i n_i V_{ie} (V_e - V_i)$$

P_{ei} and P_{ie} in terms of collision frequency;

$$R_\alpha = m_\alpha n_\alpha \sum_{\beta} \nu_{\alpha\beta} (V_\alpha - V_\beta)$$

where $\nu_{\alpha\beta}$ is elastic collisional frequency of particles α and β . While for one fluid MHD equation we suppose that collision between particles are frequent. So, their velocities (V_i, V_e, V_d) which show conservation of momentum i.e.

$$\sum_{\alpha} R_\alpha = 0$$

So we neglect term R_α in equation of motion.

Applying quasi neutrality condition i.e.

$$\sum n_\alpha e_\alpha = 0 \quad \text{on eqs (4) we}$$

get second term on R.H.S of eqs (4) as zero.

Also the current density \vec{J} for all species can be written as.

$$J_e = -n_e e v_e$$

$$J_i = Z_i n_i e v_i$$

$$J_d = -Z_d n_d e v_d$$

* As charges are equal (quasi neutral) and momentum is conserved.*

so total current density can be written as

$$J = J_e + J_i + J_d.$$

$$J = e (Z_i n_i v_i - n_e v_e - Z_d n_d v_d)$$

Putting these values in eqs (4) we have

$$m_d n_d \frac{dv_d}{dt} = \frac{1}{c} (J \times B) - 0 - \nabla (P_e + P_e + P_i + P_i) - \rho_d \nabla \psi = 0 \quad \text{--- (5)}$$

An adiabatic expression of pressure for non-relativistic temperature is given as follows. where $T < m_0 c^2$ i.e if temperature is less than the rest-mass of particle then it is non-relativistic and otherwise when $T > m_0 c^2$ i.e temp is greater than or equal to rest-mass of particles (e.g e^- , ions, dust etc)

then it is relativistic temperature.

So for non-relativistic temperature, adiabatic index is

$$\frac{P_\alpha}{n_\alpha^{5/3}} = c \text{ (constant)}$$

As $P_{0\alpha}$ and $n_{0\alpha}^{5/3}$ are also constant terms so we can write

$$\frac{P_{0\alpha}}{n_{0\alpha}^{5/3}} = c$$

so putting the value of c in above eqs.

$$\frac{P_\alpha}{n_\alpha^{5/3}} = \frac{P_{0\alpha}}{n_{0\alpha}^{5/3}}$$

$$\text{or } \frac{P_\alpha}{P_{0\alpha}} = \frac{n_\alpha^{5/3}}{n_{0\alpha}^{5/3}}$$

$$\text{or } P_\alpha = \left(\frac{n_\alpha}{n_{0\alpha}} \right)^{5/3} P_{0\alpha} \quad \text{--- (6)}$$

Now we want to write Pressure term in terms of temperature so we know that-

$$P = n k_B T$$

$$\text{or } P = n T$$

for α species.

| $\therefore T$ in energy units

$$P_{\alpha} = n_{\alpha} T_{\alpha}$$

Also

$$P_{0\alpha} = n_{0\alpha} T_{0\alpha}$$

Putting the values of P_{α} and $P_{0\alpha}$ in eqs (6)

$$n_{\alpha} T_{\alpha} = \left(\frac{n_{\alpha}}{n_{0\alpha}} \right)^{5/3} n_{0\alpha} T_{0\alpha}$$

or

$$T_{\alpha} = \left(\frac{n_{\alpha}}{n_{0\alpha}} \right)^{5/3} \left(\frac{n_{0\alpha}}{n_{\alpha}} \right) T_{0\alpha}$$

$$\text{or } T_{\alpha} = \left(\frac{n_{\alpha}}{n_{0\alpha}} \right)^{5/3} \left(\frac{n_{0\alpha}}{n_{\alpha}} \right)^{-1} T_{0\alpha}$$

$$\text{or } T_{\alpha} = \left(\frac{n_{\alpha}}{n_{0\alpha}} \right)^{2/3} T_{0\alpha}$$

So writing above equation for electrons, and

ions

for e^{-} →

$$T_e = \left(\frac{n_e}{n_{0e}} \right)^{2/3} T_{0e}$$

for ion

$$T_i = \left(\frac{n_i}{n_{0i}} \right)^{2/3} T_{0i}$$

Similarly Pressure eqs for electrons and ions

for e^{-}

$$P_e = \left(\frac{n_e}{n_{0e}} \right)^{5/3} P_{0e}$$

for ion

$$P_i = \left(\frac{n_i}{n_{0i}} \right)^{5/3} P_{0i}$$